(5)

$y = \left(\frac{\sin x}{1+\cos x}\right)^2 \qquad u = \frac{\sin x}{1+\cos x}$

$y = u^2 \qquad u = \frac{\sin x}{1+\cos x}$

$\frac{dy}{du} = 2u \qquad \frac{du}{dx} = \frac{(1+\cos)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$

$2\left(\frac{\sin x}{1+\cos x}\right)\left(\frac{1}{1+\cos x}\right) \qquad \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$\frac{2\sin x}{(1+\cos x)^2} \qquad \frac{(\cos x + 1)}{(1+\cos x)^2}$

$\frac{1}{1+\cos x}$

(11)

$s = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t$

$\frac{4}{\cancel{3}\pi}\cos(3t)\cancel{3} + \frac{4}{\cancel{5}\pi}(-\sin(5t))\cancel{5}$

$\frac{4}{\pi}\cos(3t) - \frac{4}{\pi}\sin(5t)$

30 $y'' =$

$y = \cot x$

$y' = -\csc^2 x$

$y' = -(\csc x)^2$

$y'' = -2\csc x\,(-\csc x \cot x)$

$2\csc^2 x \cot x$

33 $(f \circ g)'$

$f(u) = u^5 + 1$ $u = g(x) = \sqrt{x}$ $x = 1$

$y = u^5 + 1$ $u = \sqrt{x}$ $x^{\frac{1}{2}}$

$\frac{1}{2}x^{-\frac{1}{2}}$

$\frac{dy}{du} = 5u^4$ $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ $\frac{1}{2\sqrt{x}}$

$f(g(x)) = \sqrt{x}^5 + 1$

$x^{\frac{5}{2}} + 1$

$5(\sqrt{x})^4 \cdot \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{5}{2}x^{\frac{3}{2}}$

$\frac{5x^2}{2\sqrt{x}}$ $\boxed{\frac{5}{2}x^{\frac{3}{2}}}$

$f'(1) = \frac{5}{2}$

$\frac{5}{2}$

$(f \circ g)'$

(37) $f(u) = \frac{2u}{u^2+1}$ $u = g(x) = 10x^2 + x + 1$ $x = 0$

$y = \frac{2u}{u^2+1}$ $u = 10x^2 + x + 1$

$\frac{dy}{du} = \frac{(u^2+1)(2) - 2u(2u)}{(u^2+1)^2}(20x+1)$

$\frac{dy}{du} = \frac{2u^2+2-4u^2}{(u^2+1)^2}(20x+1)$

$= \frac{-2u^2+2}{(u^2+1)^2}(20x+1)$ $u = 10x^2 + x + 1$

$x = 0$

$= \frac{-2(1)^2+2}{(1+1)^2}(1)$ $u(0) = 1$

$\frac{-2+2}{4}(1) = 0$

EXAMPLE 7 Finding Slope

(a) Find the slope of the line tangent to the curve $y = \sin^5 x$ at the point where $x = \pi/3$.

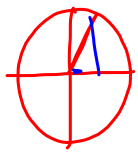
(b) Show that the slope of every line tangent to the curve $y = 1/(1 - 2x)^3$ is positive.

$y = (\sin x)^5$

$\frac{dy}{dx} = 5(\sin x)^4(\cos x) \Big|_{x=\frac{\pi}{3}}$

$5\left(\sin \frac{\pi}{3}\right)^4 \cos \frac{\pi}{3}$

$5\left(\frac{\sqrt{3}}{2}\right)^4\left(\frac{1}{2}\right)$



$\frac{5 \cdot 9}{2^5}$

$\frac{5 \cdot (\sqrt{3})^4 (1)}{2^4 \cdot 2}$

$\boxed{\frac{45}{32}}$

Multiple Choice Which of the following is dy/dx if $y = \cos^2(x^3 + x^2)$?

(A) $-2(3x^2 + 2x)$

(B) $-(3x^2 + 2x)\cos(x^3 + x^2)\sin(x^3 + x^2)$

(C) $-2(3x^2 + 2x)\cos(x^3 + x^2)\sin(x^3 + x^2)$

(D) $2(3x^2 + 2x)\cos(x^3 + x^2)\sin(x^3 + x^2)$

(E) $2(3x^2 + 2x)$

Multiple Choice Which of the following is dy/dx if $y = \tan(4x)$? E

(A) $4\sec(4x)\tan(4x)$ (B) $\sec(4x)\tan(4x)$ (C) $4\cot(4x)$

(D) $\sec^2(4x)$ (E) $4\sec^2(4x)$

2. Which of the following gives $\frac{dy}{dx}$ if $y=\cos^3(3x-2)$?

(A) $-9\cos^2(3x-2)\sin(3x-2)$

(B) $-3\cos^2(3x-2)\sin(3x-2)$

(C) $9\cos^2(3x-2)\sin(3x-2)$

(D) $-9\cos^2(3x-2)$

(E) $-3\cos^2(3x-2)$

56. ***Working with Numerical Values*** Suppose that functions f and g and their derivatives have the following values at $x = 2$ and $x = 3$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	1/3	−3
3	3	−4	2π	5

Evaluate the derivatives with respect to x of the following combinations at the given value of x.

(a) $2f(x)$ at $x = 2$
(b) $f(x) + g(x)$ at $x = 3$
(c) $f(x) \cdot g(x)$ at $x = 3$
(d) $f(x)/g(x)$ at $x = 2$
(e) $f(g(x))$ at $x = 2$
(f) $\sqrt{f(x)}$ at $x = 2$
(g) $1/g^2(x)$ at $x = 3$
(h) $\sqrt{f^2(x) + g^2(x)}$ at $x = 2$

e) $f(g(x))' = f'(g(x)) \cdot g'(x)$ $x=2$

$f'(g(2)) \cdot g'(2)$

$f'(2) \cdot (-3)$

$\left(\frac{1}{3}\right)(-3) = \boxed{-1}$

56. *Working with Numerical Values* Suppose that functions f and g and their derivatives have the following values at $x = 2$ and $x = 3$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	1/3	−3
3	3	−4	2π	5

Evaluate the derivatives with respect to x of the following combinations at the given value of x.

(a) $2f(x)$ at $x = 2$ **(b)** $f(x) + g(x)$ at $x = 3$

(c) $f(x) \cdot g(x)$ at $x = 3$ **(d)** $f(x)/g(x)$ at $x = 2$

(e) $f(g(x))$ at $x = 2$ **(f)** $\sqrt{f(x)}$ at $x = 2$

(g) $1/g^2(x)$ at $x = 3$ **(h)** $\sqrt{f^2(x) + g^2(x)}$ at $x = 2$

$\frac{1}{g^2(x)}$ $x=3$

$(g(x))^{-2}$ at $x=3$

$-2(g(x))^{-3} \cdot g'(x)$ $x=3$

$\frac{-2}{[g(3)]^3} \cdot g'(3)$

$\frac{-2}{(-4)^3} \cdot (5)$ $\frac{10}{64} = \boxed{\frac{5}{32}}$