Section 4.2 Day 1: HW 3-24 by 3's, 59,61,64

Implicit Differentiation: finding the derivative of an equation that can't be written in the form y=........ Generally, these are not functions.

Ex: $x^3+y^3-9xy=0$

$$x^3+y^3-9xy=0$$

$$3x^2+3y^2\frac{dy}{dx}-9\left[x\frac{dy}{dx}+1y\right]=0$$

$$3x^2+3y^2\frac{dy}{dx}-9x\frac{dy}{dx}-9y=0$$

$$x^2+y^2\frac{dy}{dx}-3x\frac{dy}{dx}-3y=0$$

$$y^2\frac{dy}{dx}-3x\frac{dy}{dx}=3y-x^2$$

$$\frac{\frac{dy}{dx}(y^2-3x)}{y^2-3x}=\boxed{\frac{3y-x^2}{y^2-3x}}$$

Implicit Differentiation Process

1. Differentiate both sides of the equation with respect to x
   ... treating y as a function of x and using the Chain Rule.

$\frac{dy}{dx}$

2. Collect the terms with dy/dx on one side of the equation.
3. Factor out dy/dx.
4. Solve for dy/dx.

$$\frac{d}{dx}(\cos x)^3 = 3\cos^2 x(-\sin x)$$
$$= -3\cos^2 x\sin x$$

$$\frac{d}{dx}y^3 = 3y^2\frac{dy}{dx}$$

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Example: $y^2=x$

$$y^2 = x$$

$$2y\frac{dy}{dx} = 1$$

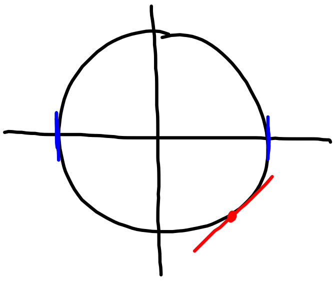
$$\frac{dy}{dx} = \frac{1}{2y}$$

Example: Finding the slope on a circle. $x^2+y^2=25$ @ (3,-4)

$$x^2+y^2=25 \qquad \frac{dy}{dx}=?$$

$$2x+2y\frac{dy}{dx}=0$$

$$2y\frac{dy}{dx}=-2x$$



$$\frac{dy}{dx}=\frac{-2x}{2y}$$

$$\frac{dy}{dx}=-\frac{x}{y}\Big|_{(3,-4)}=\frac{-3}{-4}=\boxed{\frac{3}{4}}$$

$-\frac{x}{y}$ not defined when $y=0$

Show that the slope dy/dx is defined @ every point on the previous example: 2x/(2-cosy.)

$$\frac{2x}{2-\cos y}$$

undefined

$2-\cos y=0$

$\cos y \neq 2$

never

Example: Find the tangent and normal to an ellipse $x^2-xy+y^2=7$ @ (-1,2).

$$x^2 - x\cdot y + y^2 = 7$$

$$2x - \left[x\frac{dy}{dx} + 1y\right] + 2y\frac{dy}{dx} = 0$$

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} - x\frac{dy}{dx} = y - 2x$$

$$\frac{\frac{dy}{dx}(2y-x)}{2y-x} = \frac{y-2x}{2y-x}\Bigg|_{\boxed{(-1,2)}}$$

$$\frac{2-2(-1)}{2(2)+1} = \frac{4}{5}$$

tanget

tangent

$$y-2 = \frac{4}{5}(x+1)$$

normal

$$y-2 = -\frac{5}{4}(x+1)$$

Example: $2y=x^2 + \sin y$

$$2y = x^2 + \sin y$$

$$2\frac{dy}{dx} = 2x + \cos y\frac{dy}{dx}$$

$$2\frac{dy}{dx} - \cos y\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(2-\cos y) = 2x$$

$$\boxed{\frac{dy}{dx} = \frac{2x}{2-\cos y}}$$